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EFFECT OF PRESSURE GRADIENT IN FORCED CONVECTION FILM CONDENSATION ON A HORIZONTAL TUBE

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Abstract—In earlier theoretical studies of film condensation from a vapour flowing over a horizontal tube, the pressure gradient, arising from the flow of vapour over the curved surface, has been omitted in the momentum balance for the condensate film. This has been included in the present work and shown to lead to higher heat-transfer coefficients over the forward half of the tube. It is shown further that, under certain circumstances, a solution (calculation of the condensate film thickness around the tube) is not possible for the lower part of the tube where the surface shear stress and pressure gradient act in opposite directions. Specifically, for the problem considered [vertical vapour downflow, 'potential' flow assumed outside the vapour boundary layer, asymptotic (infinite suction) approximation used for surface shear stress] an infinite rate of increase of film thickness with angle is encountered at some location on the downstream half of the tube when $\rho_g U_\infty^2/\rho g d > 1/8$. This may lead to instability of the laminar condensate film followed by some degree of waviness, turbulence, or conceivably, removal of condensate from the tube into the vapour stream. All of these possibilities would lead to enhanced heat transfer over the rear part of the tube. When $\rho_g U_\infty^2/\rho g d < 1/8$ it is found that the increase in heat transfer for the forward half of the tube is almost balanced by a decrease for the rear half, so that the mean Nusselt number for the tube is very close to that found when the pressure gradient is neglected. Numerical solutions have been obtained for wide ranges of the relevant dimensionless parameters and used to obtain accurate expressions for: (a) the mean heat-transfer coefficient for the whole tube when the pressure gradient term is unimportant, (b) the angle, ϕ_c, at which $d\delta/d\phi$ becomes infinite when $\rho_g U_\infty^2/\rho g d > 1/8$ and (c) the mean heat-transfer coefficient up to ϕ_c. A conservative equation for estimating the mean heat-transfer coefficient for the whole tube is also given and compared with available experimental data.

NOMENCLATURE

d	tube diameter
F	$\mu h_{fg} d g / U_\infty^2 k \Delta T$
G	$(\Delta T k / \mu h_{fg})(\rho\mu/\rho_g \mu_g)^{1/2}$
g	specific force of gravity
h_{fg}	specific enthalpy of evaporation
k	thermal conductivity of condensate
m	condensation mass flux
Nu	Nusselt number for the whole tube, $Qd/\Delta Tk$
$Nu_{\pi/2}$	mean Nusselt number for the upper half of the tube, $Q_{\pi/2} d/\Delta Tk$
Nu_{ϕ_c}	mean Nusselt number up to ϕ_c, $Q_{\phi_c} d/\Delta Tk$
P	$\rho_g h_{fg} \nu/\Delta Tk$
p	pressure
Q	mean heat flux for the whole tube
$Q_{\pi/2}$	mean heat flux for the upper half of the tube
Q_{ϕ_c}	mean heat flux up to ϕ_c
Re	Reynolds number for single phase flow over a tube based on free-stream approach velocity and tube diameter
$\tilde{Re}$	'two-phase' Reynolds number, $U_\infty \rho d/\mu$
r	tube radius
ΔT	vapour-to-surface temperature difference
U_∞	free-stream vapour velocity
U_ϕ	tangential velocity at 'edge' of vapour boundary layer
u	tangential velocity in condensate film
v_o	radial outward velocity at tube surface in single-phase flow
y	radial distance from tube surface.

Greek symbols

β	suction parameter, $-(v_o/U_\infty) Re^{1/2}$
δ	condensate film thickness
δ^*	$\delta(U_\infty/r\nu)^{1/2}$
θ	$F/8P = \rho g d/8\rho_g U_\infty^2$
ν	μ/ρ
μ	viscosity of condensate
μ_g	viscosity of vapour
ρ	density of condensate
ρ_g	density of vapour
τ	shear stress
τ_δ	shear stress at condensate surface
ϕ	angle (see Fig. 1)
ϕ_c	angle at which $d\delta^*/d\phi \to \infty$
ϕ_s	angle at which vapour boundary layer separates.

1. INTRODUCTION

THE PROBLEM depicted in Fig. 1 has received considerable attention in recent years. A review of the current position is given in ref. [1]. With the exception that the pressure term in the momentum balance for the condensate film is neglected in earlier studies, it may be said that the heat transfer for the upper half of the

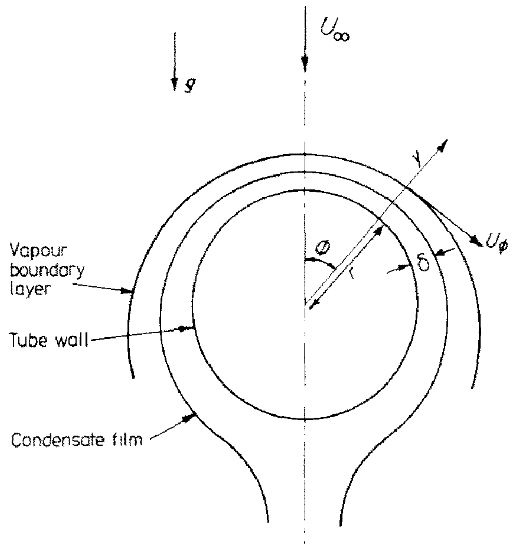

FIG. 1. Physical model.

condenser tube can now be calculated with good accuracy. Difficulties arising from the possibility of separation of the vapour boundary layer, render the calculation for the lower half of the tube less certain. Since most of the heat transfer takes place on the upper part of the tube this is not too serious and, as seen in ref. [1], three different methods of treating the condensate surface shear stress problem lead to very similar values of the mean Nusselt number for the whole tube. This may be satisfactorily represented by the equation given by Fujii *et al.* [2]:

$$Nu\,\widetilde{Re}^{-1/2} = 0.9(1+G^{-1})^{1/3} \times \{1+0.421F(1+G^{-1})^{-4/3}\}^{1/4}. \quad (1)$$

For large values of G, equation (1) becomes

$$Nu\,\widetilde{Re}^{-1/2} = 0.9(1+0.421F)^{1/4}. \quad (2)$$

Equation (2) is in close agreement with an earlier expression given by Shekriladze and Gomelauri [3], who used the asymptotic value of the surface shear stress (strictly valid only for infinite condensation rate):

$$Nu\,\widetilde{Re}^{-1/2} = 0.64\{1+(1+1.69F)^{1/2}\}^{1/2}. \quad (3)$$

For values of $G > \sim 1$ (which covers the bulk of the experimental data) the difference between equations (1)–(3) is of similar magnitude to typical scatter of the data.

The present investigation was prompted by the fact that significant discrepancies exist between measured values of Nu and those given by equations (1)–(3) (see ref. [1]). In particular, measurements for steam at high velocity [4–6] give lower heat-transfer coefficients, while recent data for R-113 [7, 8] give heat-transfer coefficients higher than the theoretical values.

2. ANALYSIS

As in the earlier studies the basic assumptions of the simple Nusselt theory are adopted [9] with the exception that the shear stress at the condensate surface is not neglected. In the present work, the circumferential variation of pressure in the condensate film is also included. A momentum balance for an element in the condensate film then gives

$$\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\rho g \sin\phi-\frac{1}{r}\frac{\mathrm{d}p}{\mathrm{d}\phi}=0, \quad (4)$$

which, with boundary conditions:

$$y=0,\quad u=0;$$
$$y=\delta,\quad \tau=\tau_\delta;$$

may be integrated to give

$$u=\frac{1}{\mu}\left\{\tau_\delta y-\left(\rho g\sin\phi-\frac{1}{r}\frac{\mathrm{d}p}{\mathrm{d}\phi}\right)\left(\frac{y^2}{2}-\delta y\right)\right\}. \quad (5)$$

Since the primary objective here is to investigate the effect of the pressure term, the simpler Shekriladze–Gomelauri [3] model is adopted, i.e.

$$\tau_\delta = mU_\phi, \quad (6)$$

the asymptotic value ($m \to \infty$), with $U_\phi \gg u_{y=\delta}$.

As indicated above, this simplification (which avoids the necessity of considering details of the vapour flow in order to match the interface shear stress at the vapour–liquid interface) is satisfactory for $G > \sim 1$ which covers most of the data. Later a small correction is indicated for lower values of G.

As in the earlier studies, potential flow outside the vapour boundary layer is considered so that

$$U_\phi = 2U_\infty \sin\phi, \quad (7)$$

and

$$\frac{\mathrm{d}p}{\mathrm{d}\phi} = -2\rho_g U_\infty^2 \sin 2\phi. \quad (8)$$

As in the Nusselt theory, a mass balance for the condensate film gives

$$m=\frac{\rho}{r}\frac{\mathrm{d}}{\mathrm{d}\phi}\left\{\int_0^\delta u\,\mathrm{d}y\right\} \quad (9)$$

and an energy balance gives

$$m = k\Delta T/\delta h_{fg}. \quad (10)$$

When equations (6)–(8) are used to eliminate τ_δ and $\mathrm{d}p/\mathrm{d}\phi$ from equation (5), the integral in equation (9) may be evaluated and the resulting value of m used in equation (10). With some re-arrangement the resulting differential equation for the condensate film thickness is

$$\frac{\mathrm{d}}{\mathrm{d}\phi}\left\{\delta^*\sin\phi+\frac{\delta^{*3}}{3} \times\left(2P\sin 2\phi+\frac{F}{2}\sin\phi\right)\right\}=\frac{1}{\delta^*}, \quad (11)$$

where

$$\delta^* = \delta(U_\infty/r\nu)^{1/2}, \quad (12)$$

$$P = \rho_g h_{fg}\nu/k\Delta T, \quad (13)$$

$$F = 2rgh_{fg}\mu/U_\infty^2 k\Delta T, \quad (14)$$

with the boundary condition

$$d\delta^*/d\phi = 0 \quad \text{at} \quad \phi = 0. \tag{15}$$

The first term inside the derivative in equation (11) results from the surface shear stress while the term involving P is due to the inclusion of the pressure gradient. When both of these terms are omitted, equation (11) reduces to the simple Nusselt problem.

Before proceeding to obtain solutions of equation (11) and thence to calculate the heat transfer for the tube, two points may be noted. First the possibility that the surface velocity gradient may go to zero is considered. This would lead to re-circulation or 'separation' of the flow in the condensate film. $(\partial u/\partial y)_{y=0}$ may be obtained from equation (5) and, using equations (6)–(8), it may be shown that the condition

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} \leqslant 0, \tag{16}$$

becomes

$$1+\frac{\delta^{*2}}{2}\left(\frac{F}{2}+4P\cos\phi\right) \leqslant 0. \tag{17}$$

Evidently condition (17) can only be satisfied for $\phi > \pi/2$ and when $P > F/8$. Secondly, inspection of equation (11) shows that $d\delta^*/d\phi$ becomes infinite when

$$1+\delta^{*2}\left(\frac{F}{2}+4P\cos\phi\right) = 0. \tag{18}$$

As in the case above equation (18) can only be satisfied for $\phi > \pi/2$ and again when $P > F/8$. Clearly the condition, equation (18), will be met at a smaller value of ϕ than that of equation (17). Since for $P > F/8$, solutions will not be possible beyond ϕ_c, the value of ϕ satisfying equation (18), the condition and associated problems given by equations (16) and (17) do not arise.

3. SOLUTIONS

As indicated above, solutions of equation (11) will only be possible for the whole tube when $P < F/8$. For this case, the dependence of δ on ϕ, for $0 < \phi < \pi$, may be determined numerically for various values of F and P, and the mean heat flux

$$Q = \frac{1}{\pi}\int_0^\pi \frac{k\Delta T}{\delta}\,d\phi, \tag{19}$$

and Nusselt number

$$Nu = \frac{Q}{\Delta T}\frac{d}{k}, \tag{20}$$

may be evaluated.

With the dimensionless film thickness defined in equation (12), equations (19) and (20) give

$$Nu\,\tilde{R}e^{-1/2} = \frac{2^{1/2}}{\pi}\int_0^\pi \frac{d\phi}{\delta^*}. \tag{21}$$

When $P \geqslant F/8$, it will only be possible to obtain solutions for $\phi < \phi_c$ where $\phi_c \geqslant \pi/2$.

Numerical solutions of equation (11) have been obtained for $F = 10^3, 10^2, 10, 1, 10^{-1}, 10^{-2}, 10^{-3}$ and 0 and for $P = 0, 0.001, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1, 2, 5$ and 10. These values extend beyond the ranges of F and P found in practice. Specimen film thickness profiles are shown in Figs. 2(a)–(c) for cases where $P = 0$, $P < F/8$ and $P > F/8$.

3.1. $P = 0$

This is the case examined by Shekriladze and Gomelauri [3] where the pressure gradient term is omitted and for which equations (2) and (3) are approximate expressions for the mean Nusselt number. Equations (2) and (3), when compared with the present solutions, were found to have maximum errors in $Nu\,\tilde{R}e^{-1/2}$ of around 7 and 2%, respectively. Alterna-

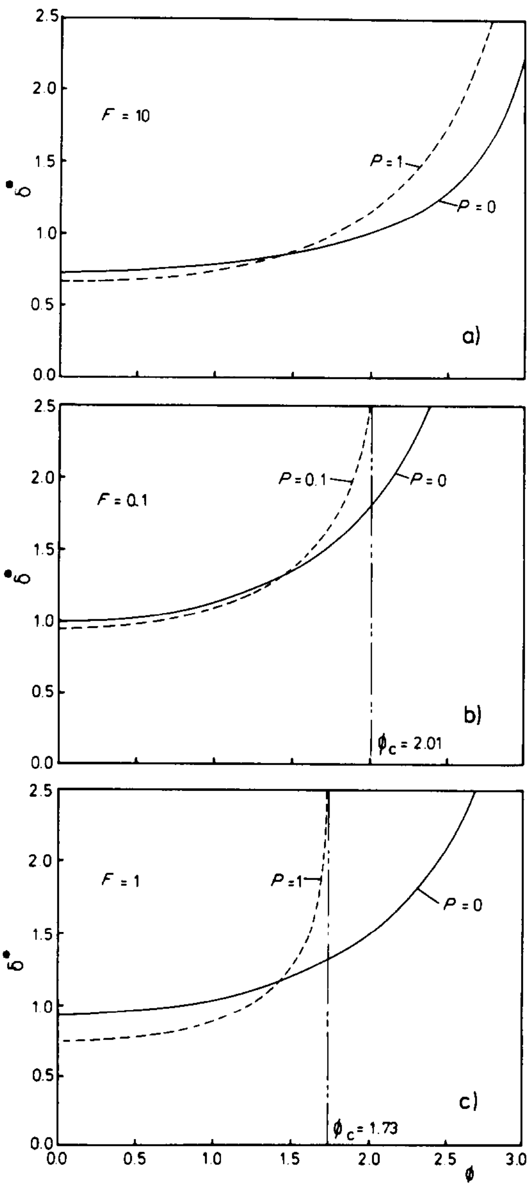

FIG. 2. Calculated thickness of condensate film.

tive expressions for $Nu\,\tilde{Re}^{-1/2}$, involving adjustable parameters, were considered. Values of the parameters were chosen so as to minimize the sum of squares of residuals of $Nu\,\tilde{Re}^{-1/2}$. Equation (22)

$$Nu\,\tilde{Re}^{-1/2} = \frac{0.9+0.728F^{1/2}}{(1+3.44F^{1/2}+F)^{1/4}}, \quad (22)$$

satisfies the zero and infinite velocity asymptotes ($F \to \infty$, $F \to 0$) and gives values of $Nu\,\tilde{Re}^{-1/2}$ to within 0.4% of the numerically obtained values for all F.

Following Fujii *et al.* [2] equation (22) may be modified to include a (generally small) correction for the fact that the asymptotic shear stress expression [equation (6)] has been used, thus

$$Nu\,\tilde{Re}^{-1/2} = \frac{0.9(1+G^{-1})^{1/3}+0.728F^{1/2}}{(1+3.44F^{1/2}+F)^{1/4}}. \quad (23)$$

3.2. $0 < P < F/8$

It may be seen from Fig. 2(a) that when the pressure term is included in the analysis (i.e. $P \neq 0$) the condensate film is thinner for the upper part of the tube and thicker for the lower part. For all of the present solutions the mean Nusselt number for the whole tube was generally within about 1% of that found when the pressure gradient term was omitted i.e. the case when $P = 0$. Only for P close to $F/8$ and when $F > 1$ was the mean Nusselt number significantly different from (and less than) that for the $P = 0$ case, the maximum discrepancy being around 5%. Thus, for practical purposes, equations (22) and (23) should be adequate when $P < F/8$.

3.3. $P \geqslant F/8$

In this case solutions cannot be obtained for $\phi > \phi_c$ as shown in Figs. 2(b) and (c). Attention is first directed to the upper half of the tube, $0 < \phi < \pi/2$, for which solutions can be obtained in all cases and where the possibility of vapour boundary-layer separation, excluded from the present analysis, should not arise. Comparisons between these results and experiments using a condenser tube with the lower half well insulated, might be used (in future studies) to establish the general validity of the theory. The results are also used below to provide a basis for a conservative design equation using accurate values for the heat transfer for the upper half of the tube and neglecting heat transfer for the lower half.

For comparison purposes in future studies, values of ϕ_c have also been obtained and the total heat transfer for the tube surface up to ϕ_c has been determined to provide a basis for a (future) study incorporating a model for the lower portion of the tube ($\phi > \phi_c$).

The mean heat flux for the upper half of the tube is given by

$$Q_{\pi/2} = \frac{2}{\pi}\int_0^{\pi/2} \frac{k\Delta T}{\delta}\,d\phi, \quad (24)$$

and the corresponding Nusselt number by

$$Nu_{\pi/2} = \frac{Q_{\pi/2}}{\Delta T}\frac{d}{k}, \quad (25)$$

so that

$$Nu_{\pi/2}\,\tilde{Re}^{-1/2} = \frac{2^{3/2}}{\pi}\int_0^{\pi/2}\frac{d\phi}{\delta^*}. \quad (26)$$

Values of $Nu_{\pi/2}$ at relatively high values of P/F are significantly greater than the corresponding values for $P = 0$ (i.e. when the pressure gradient term is omitted). For example when $F = 0.01$ and $P = 1$ the mean Nusselt number for the upper half of the tube is 24% higher than for the same value of F when $P = 0$. When the pressure gradient term is omitted ($P = 0$), $Nu_{\pi/2}\,\tilde{Re}^{-1/2}$ is given,† to within 0.6% by

$$Nu_{\pi/2}\,\tilde{Re}^{-1/2} = \frac{1.273+0.866F^{1/2}}{(1+3.51F^{0.53}+F)^{1/4}}. \quad (27)$$

When the pressure gradient term is included, for the case where the vapour shear force overwhelms gravity ($F \to 0$), the mean Nusselt number for the upper half of the tube is given† to within 0.33% by

$$Nu_{\pi/2}\,\tilde{Re}^{-1/2} = 1.273(1+1.81P)^{0.209}. \quad (28)$$

As with equation (22), equation (28) may be modified to include the (generally small) correction for the fact that the asymptotic shear stress expression has been used, thus:

$$Nu_{\pi/2}\,\tilde{Re}^{-1/2} = 1.273(1+1.81P)^{0.209} \times (1+G^{-1})^{1/3}. \quad (29)$$

Equations (27) and (29) may be used as a basis for obtaining a conservative estimate of the Nusselt number for the whole tube, thus:

$$Nu\,\tilde{Re}^{-1/2} = \frac{0.64(1+1.81P)^{0.209}(1+G^{-1})^{1/3}+0.728F^{1/2}}{(1+3.51F^{0.53}+F)^{1/4}}. \quad (30)$$

At low vapour velocity ($F \to \infty$) equation (30) blends with the Nusselt result. For the high velocity extreme ($F \to 0$), the Nusselt number given by equation (30) is that obtained when the heat transfer for the upper half of the tube is given by equation (29) and that for the lower half is neglected.

In Fig. 3 equation (30) is compared with equation (1) for a typical value of G and for a high value of P such as may be obtained for a refrigerant, and a relatively low value of P representative of the steam data. For the low value of P the enhancement of the heat transfer for the forward half of the tube, due to inclusion of the pressure term, is slight and the neglect of heat transfer for the rear half results in equation (30) predicting a smaller Nusselt number than equation (1). For the higher value of P,

† The procedure for obtaining equations (27) and (28) from the numerical solutions was the same as that described in connection with equation (22).

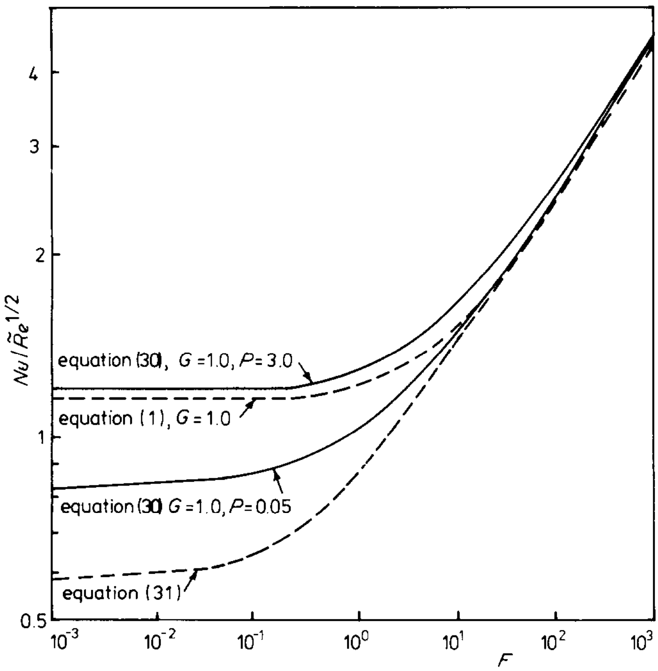

FIG. 3. Comparison of equations (1), (30) and (31).

equation (30) gives a slightly higher Nusselt number than equation (1) notwithstanding the neglect, in equation (30), of heat transfer for the rear half of the tube. Values of $Nu\ \tilde{Re}^{-1/2}$ given by equation (30) are compared, in Figs. 4(a)–(d) with observations for steam, and in Figs. 5(a)–(c) with observations for R-21 and R-113. With the exception of the high velocity data for steam [4–6] the agreement is generally good or equation (30) is somewhat conservative (e.g. R-113 data refs. [7, 17, 18]) as anticipated.

A commonly used earlier conservative estimate [10] based on the Shekriladze model [3] gives:

$$Nu\ \tilde{Re}^{-1/2} = 0.416\{1+(1+9.47F)^{1/2}\}^{1/2}. \quad (31)$$

As discussed in ref. [1] equation (31) is, from a theoretical standpoint, three-fold conservative. Equation (31) is included in Fig. 3 where it is seen to give significantly lower Nusselt numbers than equation (30).

3.4. *Values of* ϕ_c

The numerical solutions for cases where $P > F/8$ were used to determine values of ϕ_c. Two adjustable parameters, in a suitable empirical expression for ϕ_c, were determined by minimizing the sum of squares of residuals of ϕ_c to obtain the result:

$$\phi_c = \cos^{-1}\{-(1+21.5\theta P^{1.05})/(1+21.5P^{1.05})\}, \quad (32)$$

when

$$\theta = \frac{F}{8P} = \frac{\rho g d}{8\rho_g U_\infty^2} \leqslant 1. \quad (33)$$

Equation (32) satisfies the conditions: $\phi_c = \pi$, both when $P = 0$ and when $\theta = 1$; $\phi_c \to \pi/2$, when for finite F ($U_\infty \neq 0$), $P \to \infty$. Equation (32) agrees with the numerically obtained values of ϕ_c to within 0.6%. It should be noted that equation (32) strictly gives a lower limit for ϕ_c since the present solutions are based on the asymptotic expression [equation (6)] which underestimates the surface shear stress. The accuracy of equation (32) should increase with increasing values of G. Comparison of the solutions of Fujii *et al.* [2] with the present results for $P = 0$ suggests that $G > 1$ can be considered 'large' and $G > \sim 5$ is essentially infinite.

3.5. *Heat flux for tube surface up to* ϕ_c

The numerical solutions for cases where $P > F/8$ were used to determine values of the mean heat flux for $0 < \phi < \phi_c$, i.e. Q_{ϕ_c}. The corresponding Nusselt number $Q_{\phi_c}d/k\Delta T$ is given by

$$Nu_{\phi_c} = \frac{d}{\phi_c}\int_0^{\phi_c}\frac{d\phi}{\delta}, \quad (34)$$

so that, with equation (12), one obtains

$$Nu_{\phi_c}\ \tilde{Re}^{-1/2} = \frac{2^{1/2}}{\phi_c}\int_0^{\phi_c}\frac{d\phi}{\delta^*}. \quad (35)$$

The numerical solutions showed that $Nu_{\phi_c}\ \tilde{Re}^{-1/2}$ is only weakly dependent on F. In much the same way as described in connection with equations (22), (27), (28) and (32), the following equation was obtained for Nu_{ϕ_c} when $F = 0$ (i.e. infinite velocity)

$$Nu_{\phi_c}\ \tilde{Re}^{-1/2} = 2.14+0.0463P - 1.24\exp(-0.7P^{0.38}). \quad (36)$$

Equation (36) agrees with the numerically obtained values for $F = 0$ to within 1%. It may also be noted that when $P = 0$, $\phi_c = \pi$ and equation (36) gives

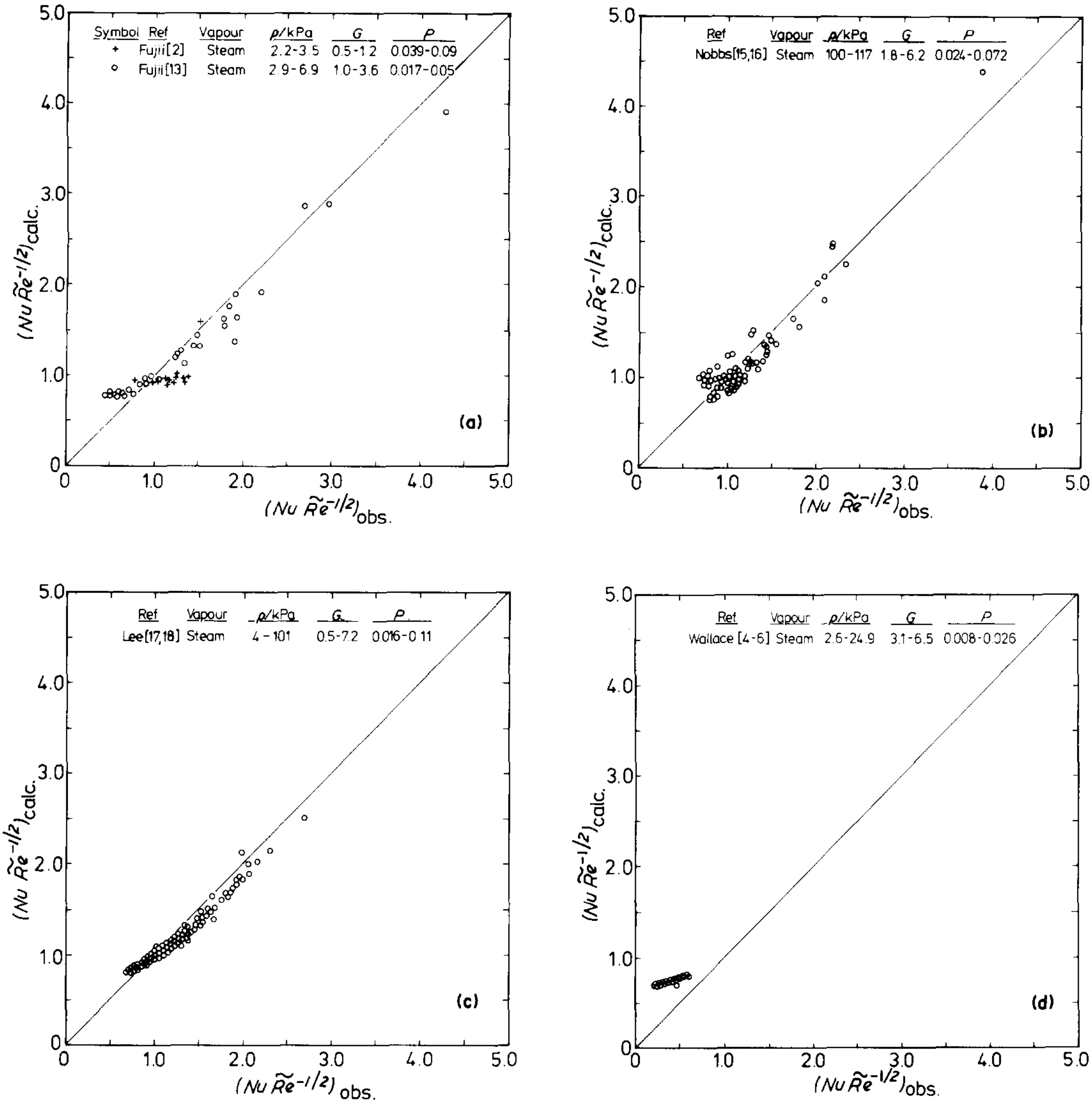

FIG. 4(a)–(d). Comparison of equation (30) with experimental data for steam.

$Nu_{\phi_c} = Nu = 0.9\,\tilde{Re}^{1/2}$ in accordance with equation (22). Owing to the weak dependence on F, equation (36) also gives the values of $Nu_{\phi_c}\tilde{Re}^{-1/2}$ to within 5%, and in most cases to within 1%, for all values of F and P covered in the present investigation. In a future 'complete' model, i.e. including the part of the film where $\phi > \phi_c$, equation (36) would give the heat transfer up to ϕ_c and the mass flow rate in the condensate film at ϕ_c.

4. CONCLUDING REMARKS

The pressure gradient term in the momentum equation has been shown to be important when

$$\theta (= F/8P = \rho g d/8\rho_g U_\infty^2)$$

is significantly less than unity. It may be seen from Table 1 that only for two of the data sets is θ always greater than unity. Owing to the higher vapour density, pressure gradient effects should become important at lower velocities for the refrigerants than for steam at comparable pressures and temperatures (see Table 1). For steam, pressure gradient effects should be more important at relatively high pressures. To date, only low pressure steam data are available owing to the large mass flow requirement to obtain appreciable velocities at higher pressures.

Inclusion of the pressure gradient term has two effects:

(a) it gives rise to an increase in the heat-transfer coefficient over the forward part of the tube. This effect is stronger for higher values of $P\,(=\rho_g h_{fg} v/\Delta T k)$, i.e. for the refrigerants (see Table 1), and

(b) it leads to an instability in the condensate film at some location over the rear half of the tube when $\theta < 1$. Depending on the nature of the flow beyond the instability (e.g. turbulence, waviness or removal of

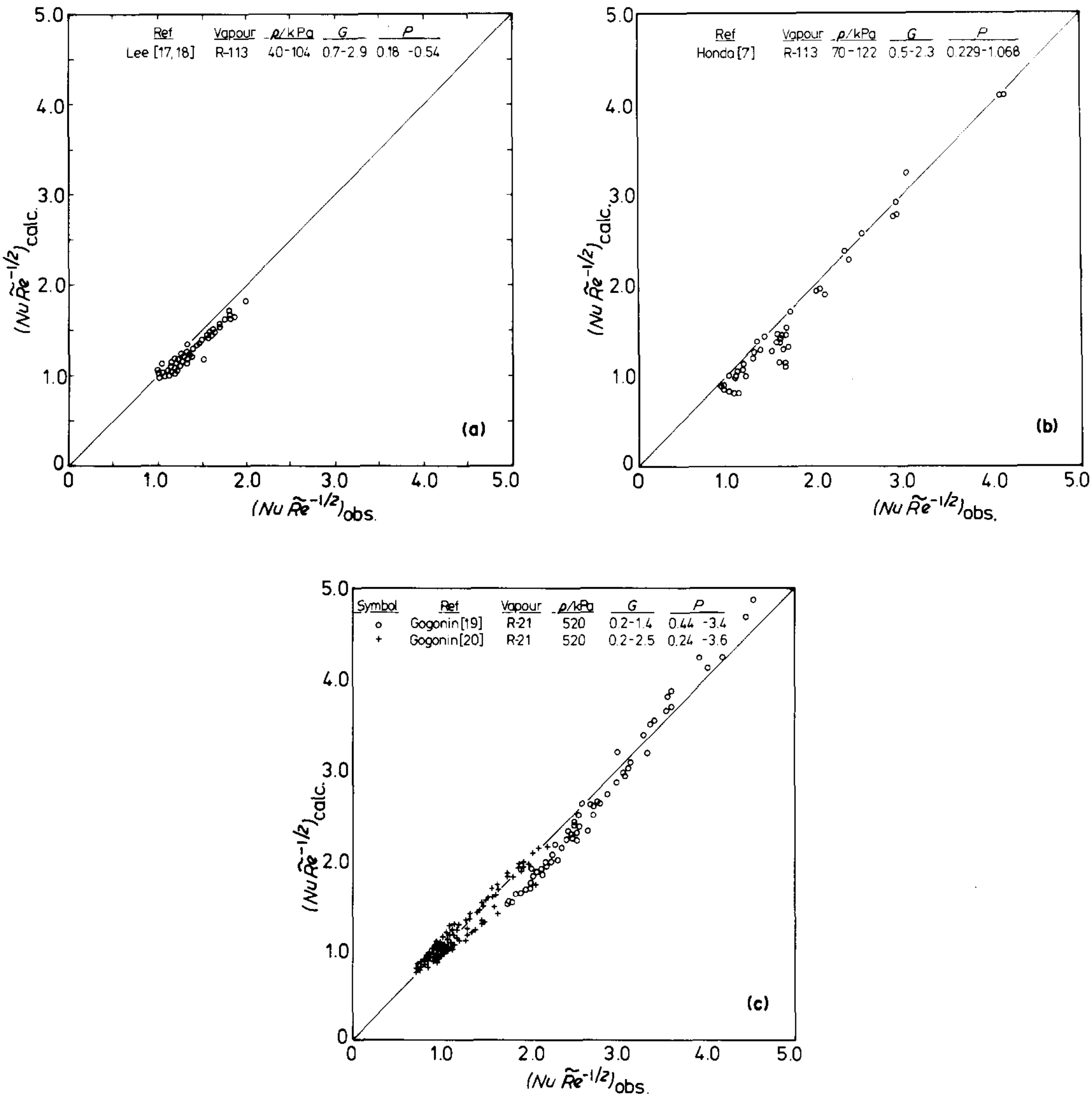

FIG. 5. Comparison of equation (30) with experimental data for R-21 and R-113.

Table 1. Approximate ranges of the parameters of experimental investigations

Ref.	Fluid	$\tilde{Re} \times 10^4$	F	G	P	$\theta = F/8P$
Fujii *et al.* [2]	steam	9.63–103	0.091–15	0.5–1.2	0.039–0.090	0.173–20.6
Fujii *et al.* [13]	steam	2.89–377	0.036–840	1.0–3.6	0.017–0.050	0.138–3607
Nobbs [15], Nobbs and Mayhew [16]	steam	0.29–52.5	0.12–3310	1.8–6.2	0.024–0.072	0.449–13 000
Lee [17], Lee and Rose [18]	steam	1.25–56.3	0.20–138	0.5–7.2	0.016–0.110	1.10–26.2
Nicol and Wallace [4, 6], Wallace [5]	steam	36.8–549	0.001–0.25	3.1–6.5	0.008–0.026	0.008–1.52
Gogonin and Dorokhov [19]	R-21	0.91–4.89	14–1922	0.2–1.4	0.440–3.400	3.90–103
Gogonin and Dorokhov [20]	R-21	0.57–48.1	0.028–94	0.2–2.5	0.240–3.600	0.012–7.85
Lee [17], Lee and Rose [18]	R-113	1.37–7.87	1.2–36	0.7–2.9	0.180–0.540	0.766–14.8
Lee *et al.* [8]	R-113	3.62–23.5	0.14–4.2	0.7–3.1	0.170–0.580	0.071–2.04
Honda *et al.* [7]	R-113	0.37–31.8	0.083–1002	0.5–2.3	0.229–1.068	0.026–296

condensate†), this could give rise to an appreciable increase in the heat-transfer coefficient over the value calculated for laminar flow when the pressure gradient term is neglected.

It should be noted that the values of ϕ_c obtained in the present work, and given with good accuracy by equation (32), are based on the asymptotic expression for the surface shear stress [equation (6)] with the 'potential flow' expression for U_ϕ [equation (7)]. Equation (32) would thus be invalid if vapour boundary-layer separation occurred at an angle ϕ_s less than ϕ_c. If separation were accompanied by a sharp pressure rise this could initiate an instability in the condensate film near ϕ_s. Alternatively if, for $\phi > \phi_s$, the pressure were to remain essentially uniform and the surface shear stress were negligible, it may be that no irregularity of the film would be seen. Lee [11] has used the method of Truckenbrodt [12], described by Fujii *et al.* [13] and discussed in refs. [1, 14], to estimate the angle at which vapour boundary-layer separation occurs. These results, strictly for flow over a cylinder with uniform suction, have been used by the present author to obtain approximate expressions for ϕ_s. Values obtained are closely approximated by

$$\phi_s = 1.76 + 0.164\beta + 0.00869\beta^2, \quad (37)$$

when adopting the method of Truckenbrodt [12] directly and by:

$$\phi_s = 2.93 - 1.02 \exp(-0.147\beta - 0.0127\beta^2), \quad (38)$$

when adopting the method of Truckenbrodt [12] as modified by Fujii *et al.* [13].

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† References to 'wrinkled film' condensation [13], 'stripping of condensate' [4, 5] and 'abrupt change of film thickness' [7], all for the rear part of the tube, may be noted.

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EFFET D'UN GRADIENT DE PRESSION EN CONVECTION FORCEE AVEC CONDENSATION EN FILM SUR UN TUBE HORIZONTAL

Résumé—Dans les études théoriques antérieures sur la condensation en film d'une vapeur sur un tube horizontal, le gradient de pression qui provient de l'écoulement de la vapeur sur la surface courbe a été omis dans le bilan de quantité de mouvement du film de condensat. Il a été inclus dans ce travail et il conduit à des coefficients de transfert thermique plus élevés sur la moitié amont du tube. On montre que, sous certaines conditions, une solution (calcul de l'épaisseur du film autour du tube) n'est pas possible pour la partie inférieure du tube où la tension de cisaillement superficielle et le gradient de pression agissent dans des directions opposées. Pour le problème considéré [écoulement vertical descendant de la vapeur, écoulement "potentiel" supposé hors de la couche limite de vapeur, approximation asymptotique (succion infinie) utilisée pour le cisaillement pariétal], un accroissement infini de l'épaisseur du film avec l'angle est rencontré à un certain endroit sur la moitié aval du tube quand $\rho_g U_\infty^2/\rho g d > 1/8$. Ceci peut conduire à une instabilité du film laminaire de condensat suivie par une ondulation, la turbulence ou l'enlèvement du condensat dans le courant de vapeur. Chacune de ses possibilités conduit à un accroissement du transfert thermique sur la partie arrière du tube. Quand $\rho_g U_\infty^2/\rho g d < 1/8$, on trouve que l'accroissement de transfert thermique sur la moitié avant du tube est à peu près compensée par une diminution sur la face arrière, de telle sorte que le nombre de Nusselt moyen sur le tube est très proche de celui obtenu quand on néglige le gradient de pression. Des solutions numériques sont obtenues pour des domaines étendus des paramètres sans dimension et utilisées pour obtenir des expressions précises pour : (a) le coefficient moyen de transfert pour le tube entier, (b) l'angle ϕ_c où $d\delta/d\phi$ devient infini quand $\rho_g U_\infty^2/\rho g d > 1/8$ et (c) le coefficient moyen de transfert jusqu'à ϕ_c. Une équation conservative pour estimer le coefficient moyen pour le tube entier est donnée et comparée avec des données expérimentales.

DER EINFLUSS DES DRUCKGRADIENTEN BEI DER FILMKONDENSATION AN EINEM HORIZONTALEN ROHR BEI ERZWUNGENER STRÖMUNG

Zusammenfassung—In früheren theoretischen Untersuchungen der Filmkondensation eines strömenden Dampfes an einem horizontalen Rohr wurde der Druckgradient, der durch die Strömung des Dampfes über die gekrümmte Oberfläche entsteht, in der Impulsbilanz für den Kondensatfilm vernachlässigt. In der vorliegenden Arbeit wurde er berücksichtigt, und es wurde festgestellt, daß er zu höheren Wärmeübergangskoeffizienten an der vorderen Hälfte des Rohres führt. Weiter wird gezeigt, daß unter gewissen Umständen eine Lösung (Berechnung der Kondensatfilmdicke um das Rohr) für den unteren Teil des Rohres, wo die Oberflächenschubspannung und der Druckgradient in entgegengesetzten Richtungen auftreten, nicht möglich ist. Speziell für das betrachtete Problem vertikale Dampfströmung, angenommene "Potentialströmung" außerhalb der Dampfgrenzschicht, asymptotische Näherung (unendliche Absaugung) für die Oberflächenschubspannung wird ein unendlich schnelles Anwachsen der Filmdicke mit dem Winkel an irgendeiner Stelle auf der unteren Hälfte des Rohres festgestellt, sofern $\rho_g U_\infty^2/\rho g d > 1/8$ beträgt. Dies kann zur Instabilität des laminaren Kondensatfilms, verbunden mit einem beträchtlichem Grad von Welligkeit, Turbulenz oder—was auch denkbar ist—dem Abreißen von Kondensat vom Rohr in den Dampfstrom führen. Alle diese Möglichkeiten würden zu einem erhöhten Wärmeübergang an der Rückseite des Rohres führen. Wenn $\rho_g U_\infty^2/\rho g d < 1/8$ beträgt, zeigt sich, daß die Zunahme des Wärmeübergangs an der vorderen Hälfte des Rohres durch einen Rückgang an der hinteren Hälfte fast ausgeglichen wird, so daß die mittlere Nusselt-Zahl für das Rohr derjenigen bei vernachlässigtem Druckgradienten sehr nahe kommt. Numerische Lösungen wurden über große Bereiche der relevanten dimensionslosen Parameter erhalten, und es wurden exakte Ausdrücke berechnet für (a) den mittleren Wärmeübergangskoeffizienten für das ganze Rohr, wenn der Druckgradient unwichtig ist, (b) der Winkel ϕ_c, bei dem $d\delta/d\phi$ unendlich wird, wenn $\rho_g U_\infty^2/\rho g d > 1/8$ ist und (c) den mittleren Wärmeübergangskoeffizienten bis ϕ_c. Eine Gleichung zur Abschätzung des mittleren Wärmeübergangskoeffizienten für das gesamte Rohr wird ebenfalls angegeben und mit verfügbaren experimentellen Daten verglichen.

ВЛИЯНИЕ ГРАДИЕНТА ДАВЛЕНИЯ НА ПЛЕНОЧНУЮ КОНДЕНСАЦИЮ ПРИ ВЫНУЖДЕННОЙ КОНВЕКЦИИ НА ГОРИЗОНТАЛЬНОЙ ТРУБЕ

Аннотация—В ранее проводившихся теоретических исследованиях пленочной конденсации пара, обтекающего горизонтальную трубу, градиент давления, возникающий при течении пара на искривленной поверхности, не учитывался в уравнении баланса количества движения для пленки конденсата. В данной работе расчет проводился с учетом градиента давления и показано, что в результате получаются более высокие значения коэффициентов переноса тепла на передней части трубы. Кроме того показано, что при определенных условиях решение (расчет толщины пленки конденсата вокруг трубы) невозможно получить для нижней части трубы, где напряжение сдвига на стенке и градиент давления противоположны по направлению. В частности, в рассматриваемой задаче [вертикальное опускное течение пара, предположение о наличии «потенциального» течения вне пограничного слоя пара, использование асимптотического (бесконечный отсос) приближения для напряжения сдвига на стенке] неограниченный рост толщины пленки с увеличением угла стекания имеет место в некоторой области центральной части трубы по течению, когда $\rho_g U_\infty^2/\rho g d > 1/8$. Это может приводить к неустойчивости ламинарной пленки конденсата с появлением небольшой волнистости, турбулентности и, возможно, к уходу конденсата из трубы в поток пара. Это может вызывать интенсификацию теплопереноса на задней части трубы. Для $\rho_g U_\infty^2/\rho g d < 1/8$ установлено, что усиление теплопереноса на передней части трубы почти уравновешивается его уменьшением на задней части, так что среднее значение числа Нуссельта для трубы оказывается весьма близким к значению, найденному в том случае, когда градиент давления не учитывается. Численные решения получены для широких диапазонов соответствующих безразмерных параметров и использованы для вывода выражений, с помощью которых можно точно рассчитать (а) средний коэффициент теплопереноса для всей трубы, когда членом уравнения, учитывающим градиент давления, можно пренебречь, (б) угол ϕ_c, при котором отношение $d\delta/d\phi$ стремится к бесконечности при $\rho_g U_\infty^2/\rho g d > 1/8$ и (в) средний коэффициент теплопереноса вплоть до ϕ_c. Предложено также консервативное уравнение для оценки среднего значения коэффициента теплопереноса для всей трубы и проведено сравнение с имеющимися экспериментальными данными.